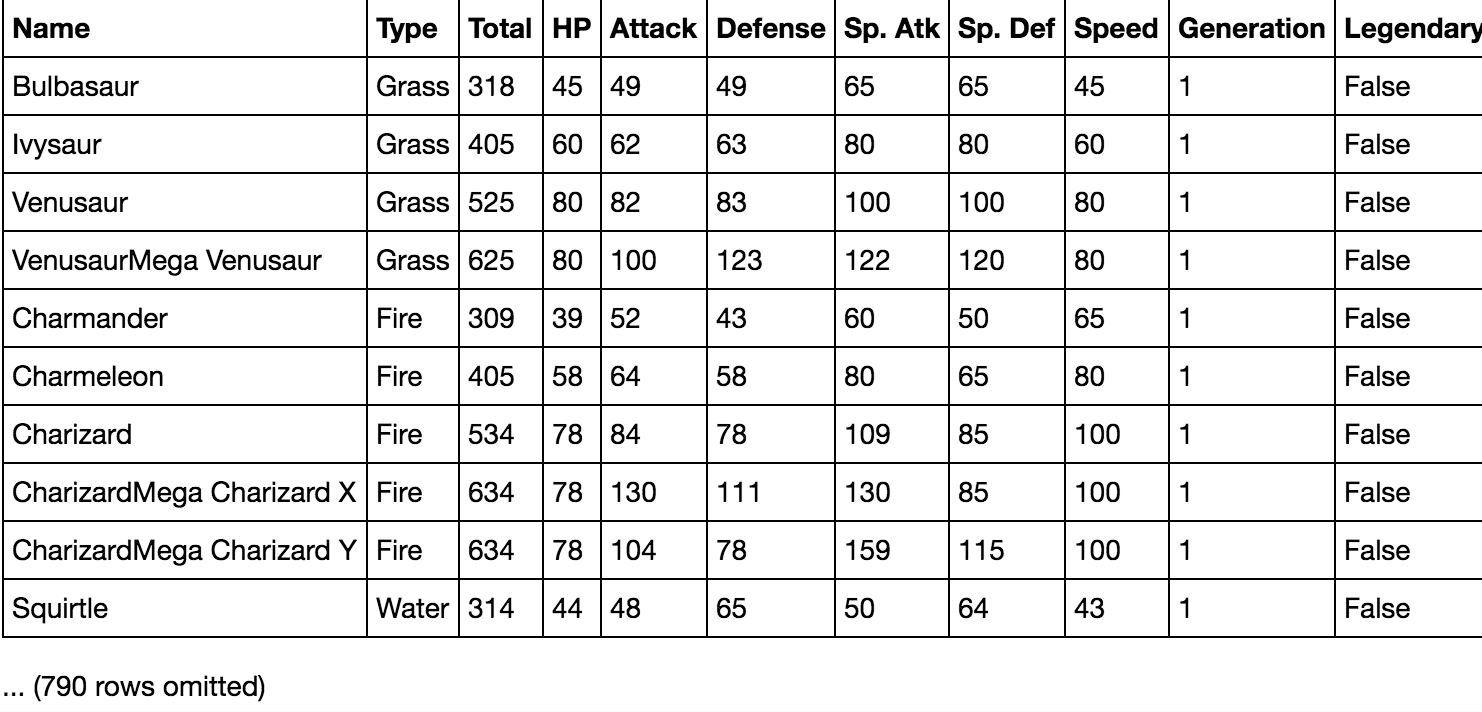
**CMPSC 5B – Spring 2023**

**Discussion: CMPSC 5A Review, Ch 1-11**

**Tables**

You are given the following table called pokemon. For the following questions, fill in the blanks.



1. Find the name of the pokemon of type Water that has the highest HP.

water\_pokemon = pokemon.\_\_\_\_\_\_\_\_\_\_\_\_\_(\_\_\_\_\_\_\_\_\_\_\_,\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_)

water\_pokemon.\_\_\_\_\_\_\_\_(\_\_\_\_\_\_\_\_\_\_,\_\_\_\_\_\_\_\_\_\_\_\_).column("Name").item(0)

2. Find the proportion of pokemon of type Fire in the dataset whose Speed is strictly less than 100.

fire\_pokemon = pokemon.\_\_\_\_\_\_\_\_\_\_\_\_\_(\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_,\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_)

fire\_pokemon.\_\_\_\_\_\_\_(\_\_\_\_\_\_\_\_,\_\_\_\_\_\_\_\_\_\_\_\_).\_\_\_\_\_\_\_\_/\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

3. Return a table containing Type and Generation that is sorted in decreasing order by the average HP for each pair of Type and Generation.

d = pokemon.\_\_\_\_\_\_\_\_\_\_\_\_(\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_,\_\_\_\_\_\_\_\_\_\_\_\_\_\_)

d.sort("HP mean",\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_).\_\_\_\_\_\_\_\_(\_\_\_\_\_\_\_\_,\_\_\_\_\_\_\_\_\_\_\_\_)

4. Find the largest difference of average HP between consecutive generations of Pokemon.

generation = pokemon.\_\_\_\_\_\_\_\_(\_\_\_\_\_\_\_\_\_\_\_\_\_\_,\_\_\_\_\_\_\_\_\_\_\_)\

.sort("Generation",descending=False)

\_\_\_\_\_\_\_\_(np.diff(\_\_\_\_\_\_\_\_\_\_\_\_\_\_.\_\_\_\_\_\_\_\_\_("HP mean")))

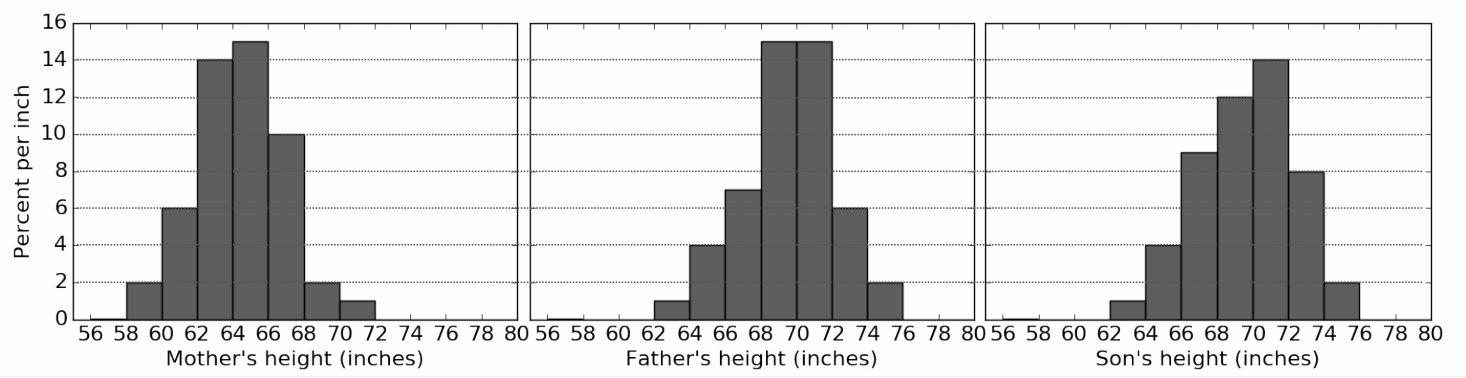
5. Return an array that contains ratios of legendary to non-legendary pokemons for each generation.

t = pokemon.\_\_\_\_\_\_\_\_\_\_\_\_\_(\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_,\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_)

ratio = t.\_\_\_\_\_\_\_\_\_\_\_\_(\_\_\_\_\_\_\_\_\_\_\_\_\_\_)/t.\_\_\_\_\_\_\_\_\_\_\_\_(\_\_\_\_\_\_\_\_\_\_\_\_)

**Histograms**

Galton measured the heights of the members of **200 families** that each included 1 mother, 1 father, and some varying number of adult sons. The three histograms of heights below depict the distributions for all mothers, fathers, and adult sons. All bars are 2 inches wide. All bar heights are integers. The heights of all people in the data set are included in the histograms.



1. Calculate each quantity described below or write *Unknown* if there is not enough information above to express the quantity as a single number (not a range). Show your work!

a. The **percentage** of mothers that are at least 58 inches but less than 62 inches tall.

b. The **percentage** of fathers that are at least 62 inches but less than 65 inches tall.

c. The **number** of sons that are at least 72 inches tall.

d. The **number** of mothers that are less than 70 inches tall.

2. If the father’s histogram were redrawn, replacing the three bins from 68-70, 70-72 and 72-to-74 with one bin from 68-to-74, what would be the height of its bar? If it’s impossible to tell, write *Unknown*.

3. The percentage of sons that are taller than all of the fathers is between \_\_\_\_\_\_\_\_\_\_\_ and \_\_\_\_\_\_\_\_\_\_\_. Fill in the blanks in the previous sentence with the smallest range that can be determined from the histograms, then explain your answer below.

**Probability**

1. A fair coin is tossed five times. Two possible sequences of results are HTHTH and HTHHH. Which sequence of results is more likely? Explain your answer and calculate the probability that each sequence appears.

2. Consider a biased coin such that the probability of getting heads is ⅕ and the probability of getting tails is 4/5. The coin is tossed 3 times. What is the probability that you get exactly 2 heads?

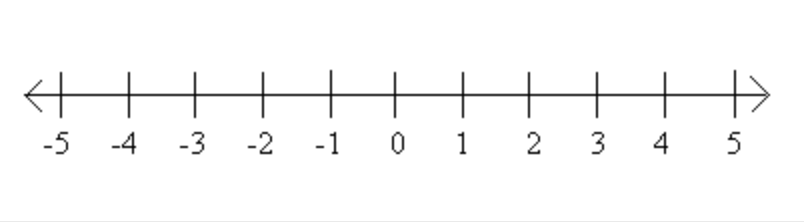
3. Once again, we toss the same coin 3 times. What is the probability I get no heads?

4. Again, we toss the same coin 3 times. What is the probability I get at least 1 heads?

*Hint: There are two ways of calculating this probability. One is significantly easier to calculate than the other.*

**Simulation and Hypothesis Testing**

Achilles the turtle sits on the number line. Achilles loves long random walks that last a total of 100 time steps. At each time step, Achilles moves based on the following scheme: He flips a coin and moves one step to the right if the coin comes up heads or one step to the left if the coin comes up tails.



1. Assuming that Achilles’ coin is fair, write a function called one\_walk that simulates one random walk of 100 time steps and returns how far from the origin Achilles ends up at the end of his walk. You may assume that Achilles always starts from the origin.

def one\_walk():

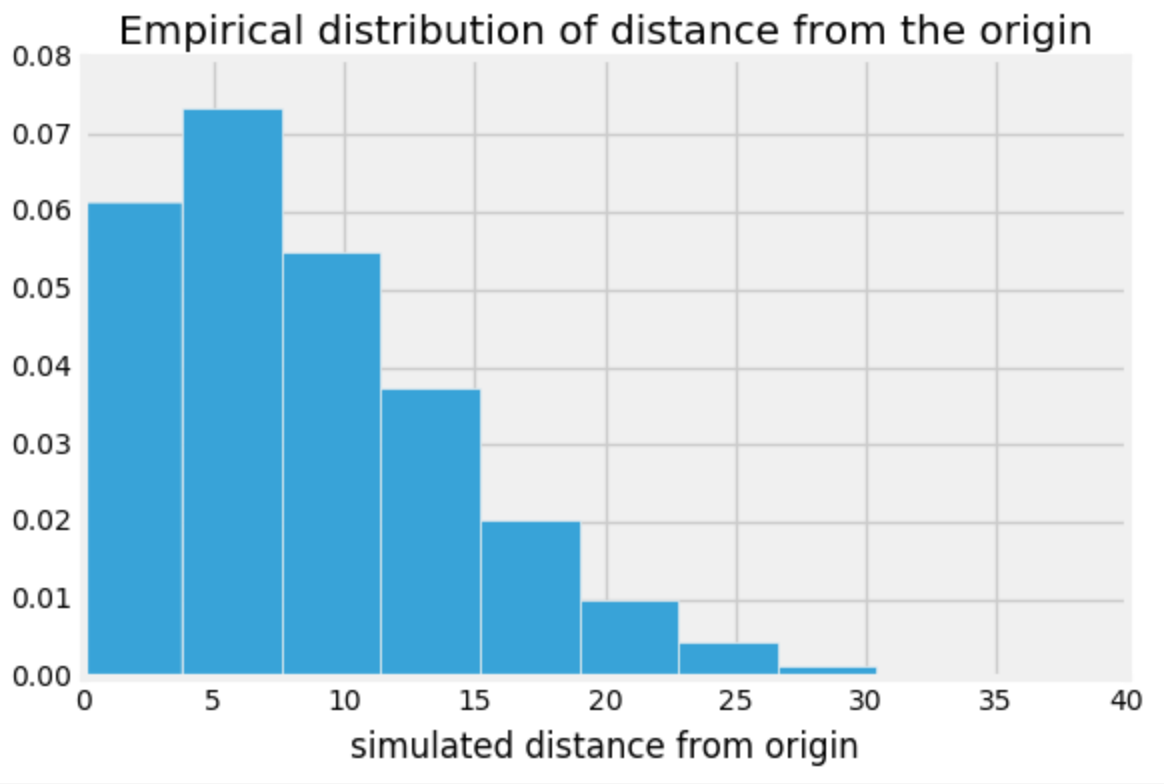
2. Assuming that Achilles’ coin is fair, we would like to simulate what would happen if Achilles took 10000 different random walks. Complete the simulation below and keep track of how far Achilles ends up from the origin in each of his walks in an array called distances. The histogram shown below is an example of a histogram plotted from distances.

distances = make\_array()

for i in np.arange(10000):

new\_distance = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

distances = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_



3. Achilles goes for a walk and claims that at the end of his walk, he ended up 30 steps away from the origin. You notice this is strange, so you want to run a hypothesis test to test whether or not Achilles used a fair coin. Fill in the blanks below for the null and alternative hypotheses as well as a good test statistic for this experiment.

*Hint: When considering your alternative hypothesis, note that we do not really care about whether the coin is biased towards heads or towards tails.*

**Null Hypothesis:**

**Alternative Hypothesis:**

**Test Statistic:**

4. Write the code to calculate the p-value given the test statistic listed above and using a 5% p-value cut-off. Then, describe the different conclusions that you would arrive at depending on the p-value.

*Hint: We simulated an array in part(b) of test statistics under the null hypothesis. Try to use the* distances *array.*

p\_value = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**True/False**

Respond with true or false to the following questions. If your answer is false, explain why.

1. In the U.S. in 2000, there were 2.4 million deaths from all causes, compared to 1.9 million in 1970, which represents a 25% increase. The data shows that the public’s health got worse over the period 1970-2000.

2. A company is interested in knowing whether women are paid less than men in their organization. They share *all* their salary data with you. An A/B test is the best way to examine the hypothesis that all employees in the company are paid equally.

3. Consider a randomized control trial where participants are randomly split into treatment and control groups. There will be no systematic differences between the treatment and control groups if the process is followed correctly.

4. A researcher considers the following scheme for splitting a people into control and treatment groups. People are arranged in a line and for each person, a fair, six-sided die is rolled. If the die comes up to be a 1 or a 2, the person is allocated to the treatment group. If the die comes up to be a 3, 4, 5 or 6 then the person is allocated to the control group. This is a randomized control experiment.

5. You are conducting a hypothesis test to check whether a coin is fair. After you calculate your observed test statistic, you see that its p-value is below the 5% cutoff. At this point, you can claim with certainty that the null hypothesis can not be true.

6. You roll a fair die a large number of times. While you are doing that, you observe the frequencies with which each face appears and you make the following statement: As I increase the number of times I roll the die, the probability histogram of the observed frequencies converges to the empirical histogram.

**Bonus Question:** What is the Law of Averages? Can you see why it allows us to run large scale simulations instead of trying to find exactly what the probability distribution of a test statistic is?

**Multiple Choice**

1. Gary is playing with a coin and he wants to test whether his coin is fair. His experiment is to toss the coin 100 times. He chooses the following null hypothesis:

**Null Hypothesis:** The coin is fair and any deviation observed is due to chance.

For each of the alternative hypotheses listed below, determine whether or not the test statistic is valid.

a. **Alternative Hypothesis:** The coin is biased towards heads.

**Test Statistic:** # of heads

b. **Alternative Hypothesis:** The coin is not fair.

**Test Statistic:** # of heads

c. **Alternative Hypothesis:** The coin is not fair.

**Test Statistic:** |# of heads - expected # of heads|

d. **Alternative Hypothesis:** The coin is biased towards heads.

**Test Statistic:** |# of heads - expected # of heads|

e. **Alternative Hypothesis:** The coin is not fair.

**Test Statistic:** ½ - proportion of heads

2. It is now generally accepted that cigarette smoking causes heart disease, lung cancer, and many other diseases. However, in the 1950s, this idea was controversial. The statistician and geneticist R. A. Fisher advanced the “constitutional hypothesis”, which claims there is some genetic factor that predisposes individuals to smoke as well as to die from diseases.

Suppose that Fisher was correct and there is a gene that predisposes individuals towards smoking as well as getting lung cancer. In the context of this experiment, how would you characterize this gene?

A. treatment

B. outcome

C. confounding factor

D. placebo

**Fun with Functions**

1. Write a function called compute\_pvalue that given an empirical distribution in the form of an array and the observed value of your test statistic, calculates the p-value for that test statistic. You may assume that large values of your test statistic provide evidence against the null hypothesis.

def compute\_p\_value(empirical\_dist, observed\_ts):

2. Now write a function called is\_significant that takes in an empirical distribution, the observed test statistic and a p-value cutoff, returns True if the p-value of the observed test statistic is statistically significant based on the cutoff provided and False otherwise.

*Hint: Use the function you defined in Question 1!*

def is\_significant(empirical\_dist, observed\_ts, cutoff):

return

3. Write a function called is\_prime that takes in a number n and returns True if the number is prime and False otherwise. Remember that a number is prime if it is only divisible by itself and 1. In general, we do not consider 1 to be a prime number.

*Hint: The % operator is your friend.*

def is\_prime(n):

return

**More Hypothesis Testing**

Chloe is a big fan of Trader Joes’ frozen mac n cheese, but she noticed that the cheese used in it varies from box to box. A Trader Joe’s employee provides her with some data about the 4 different cheeses used and the probability of them being used in each box:

|  |  |
| --- | --- |
| **Cheese** | **Probability** |
| Velveeta | 0.05 |
| Gruyère | 0.55 |
| Sharp Cheddar | 0.25 |
| Monterey Jack | 0.15 |

Chloe is suspicious about this distribution. After all, Velveeta is much cheaper to use than Gruyère, and she has also never bought a box that uses Gruyère. Chloe decides to buy many boxes throughout the next month and tracks the type of cheese used in each box. She uses this to conduct a hypothesis test.

1. Write the correct null hypothesis for this experiment

* Null Hypothesis:
* Alternative Hypothesis:

observed\_proportions = make\_array(0.2, 0.3, 0.45, 0.05)

employee\_proportions = make\_array(0.05, 0.55, 0.25, 0.15)

The array observed\_proportions contains the proportions of cheese that Chloe observed in 20 boxes of Mac n Cheese.

2. Chloe wants to use the mean as a test statistic, but Katherine suggests that she uses the TVD (total variation distance) instead. Which test statistic should Chloe use in this case? Briefly justify your answer. Then write a line of code to assign the observed value of the test statistic to observed\_stat.

observed\_stat = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

3. Define the function one\_simulated\_test\_stat to simulate a random sample according to the null hypothesis and return the test statistic for that sample.

def one\_simulated\_test\_stat():

sample\_prop = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

return \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

4. Chloe simulates the test statistic 10,000 times and stores the results in an array called simulated\_stats. The observed value of the test statistic is stored in observed\_stat. Complete the code below so that it evaluates to the p-value of the test:

\_\_\_\_\_\_\_\_\_\_\_\_\_(simulated\_stats \_\_\_\_\_\_\_\_\_ observed\_statistic) / \_\_\_\_\_\_\_\_\_\_\_\_\_

5. Given that the computed p-value is 0.0825, which of the following are true? Select all that may apply.

1. Using an 8% p-value cutoff, the null hypothesis should be rejected
2. Using a 10% p-value cutoff, the null hypothesis should be rejected.
3. There is an 8.25% chance that the null hypothesis is true
4. There is an 8.25% chance that the alternative hypothesis is true